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NATURE.

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Biela's Comets	May 1, 1873.
Meteors of Nov. 14	June 3, 1875.

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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph.D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

CHAPTER SECOND.

THE FIRST TREATISE ON NON-EUCLIDEAN GEOMETRY.

[Continued from the April Number.]

Is it not surprising that a book so remarkable, that it will henceforth forever mark an epoch in human thought, should have been forgotten for more than a century and a half? The first treatise on Non-Euclidean Geometry, a work of extraordinary genius, appeared in 1733, yet so far as I know, not one single copy of this wonderful book is owned on the Western Continent.

That I have had the book for a considerable time in my possession is due to the generosity of a learned prelate of Louvain, R. P. Tnirion, S. J., who lent me, across the ocean, his only copy of this rare and inestimable treasure.

I have already made the distinction between Anti-Euclideans, such as Bertrand of Geneva, Legendre, M. Vincent, Ed. T. Dixon, etc., who try to convict Euclid of imperfection by offering short proofs of his celebrated Parallel-Postulate; and the true Non-Euclideans, such as Lambert, Bolyai, Lobatschewsky, Riemann, Helmholtz, Lie, Klein, Clifford, Cayley, etc., all ardent admirers of Euclid, but makers of two companion geometries, called usually Lobatschewsky's and Riemann's.

But a whole century before Lobatschewsky, it must have been by a sort of prophetic instinct that the Italian Saccheri called his book, now ressurrected for immortality, "Euclid vindicated from every fleck." The complete title is as follows: Euclides—ab omni naevo vindicatus:—sive—conatus geometricus—quo stabiliuntur—Prima ipsa universae Geometriae Principia.—Auctore—Hieronymo Saccherio—Societatis Jesu—In Ticinensi Universitate Matheseos Professore.—Opusculum—Ex.^{mo} Senatin—Mediolanensi—Ab Auctore Dicatum.—Mediolani, MDCCXXXIII.—Ex Typographia Pauli Antonii Montani. Superiorum permissu.

This book is an intensely interesting historical study, not only in science, but in psychology, and ethics; for it is evident to any mathematician reading it with the light we now have, that Saccheri deliberåtly built up a non-Euclidean geometry, and then covered it with just enough disguise to enable it to pass the the ordeal of authorization by Ignatius Vicecomes, Provincial of the Jesuits, who certefies that it had been read by some Theologians of that society and found fit to see the light; and to enable it to receive the "*Imprimatur*" of the Senate, a Cardinal, and the Inquisitor General, Sylvester Martini, by whose order it was carefully read by the Revisor Don Gaspar, Doctor both of law and of Sacred Theology, who declared it to contain nothing against the orthodox faith.

Remember DeMorgan's saying: "As to writing another work on geometry, the middle ages would as soon have thought of composing another New Testament......his order of demonstration was thought to be necessary and founded in the nature of our minds;" and remember that Saccheri's book contains not merely "another *work* on geometry," but *another geometry*, a thought so tremendous, so unòrthodox, that its discovery in his book by these great Church Dignitaries would have doomed Saccheri to death. Perhaps an after suspicion of the truth *did* doom him to death, for the permission of the Provincial was given August 16th, 1733, and Saccheri was dead October 25th, 1733.

There exists in the library of Modena, Italy, a Manuscript Biography of Saccheri. Dr. Emory McClintock, the able President of the New York

Mathematical Society, writes of Saccheri: "He confessed to a distracting heretical tendency on his part in favor of the 'hypothesis anguli acuti,' a tendency against which, however, he kept up a perpetual struggle. After yielding so far as to work out an accurate theory anticipating Lobatschewsky's doctrine of the parallel-angle,he appears to have conquered the internal enemy abruptly, since, to the surprise of his commentator Beltrami, he proceeded to announce dogmatically that the specious 'hypothesis anguli acuti' is positively false." Of course no such confession occurs or could have occured in the book itself; for with it the book could never have been printed. If such a statement occurs in the Manuscript Biography that muſt have been the work of a personal friend written after Saccheri's death.

The sudden dogmatic assertion above mentioned does occur, first on page 70 of the work, a quarto. But this, after seventy quarto pages of rigorous logic and elegant demonstration to establish a non-Euclidean geometry, may be looked upon as something like the stucco for the king's inspection with which the immortal architect in Egypt covered the stone bearing his own name.

A whole century later, 1829, Gauss, in the "Lehr-und Lernfreiheit" of a German University, writes to his friend Bessel that he will never publish his researches on this subject, "da ich das Geschrei der Gegner scheue, wenn ich meine Ansicht ganz aussprechen wollte."

Saccheri's marvellous book says all it could have said and existed; apart from any decision of the historical question as to how consciously the Italian jesuit was in it practising the motto of his order: "The end justifies the mean's.

The expression on the title-page, "the very First Principles of Universal Geometry," strongly suggests Lobatschewsky's *Pangeometry.*"

The book opens with a dedication to the Senate of Milan, followed by a Preface to the Reader.

This, after a powerful eulogy of Euclid, begins by stating that the Parallel-Postulate has ever been a vexed question. "Though heretofore no one has doubted the truth of the proposition, yet many have maintained that it is not axiomatic. So not a few have attempted to demonstrate it from those propositions of the First Book of Euclid which precede the twenty-ninth,wherein it is first used. But since all these attempts have signally failed, many, more recently, have attacked the matter by setting up a certain new definition of parallels.

Whereas Euclid defines parallels thus 'Any straight lines, which are in the same plane, and being produced indefinitely towards both sides meet each other on neither, are parallels;' these,for the latter words substitute '*are always mutually equidistant.*' But here arises a new split.

For some,and those certainly the more acute, have endeavored to show the existence of parallel straight lines as thus defined, whence they make the transition to the proposition to be demonstrated under Euclid's own words.

But others (not without a great sin against rigorous logic) assume as given such parallel straight lines, forsooth *equidistant*, that thence they may make the step to what remains to be proved." Saccheri then goes on to say that he will not at first go into the question of the nature of those lines equidistant at all points from a certain line supposed straight, but will return to it later. "But any one can see that herein is occasion for subjecting to a rigorous examination the very first principles of universal geometry."

This is Saccheri's excuse, his plea, his defense for introducing a new kind of geometry.

"Atque hinc incipit diuturnum praelium adversus hypothesin anguli acuti."

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

13. Proposed by J. R. BALDWIN, A. M., Professor of Mathematics in the Davenport Business College, Davenport, Iowa.

A man borrowed $5000 at a western bank giving his note for $5000 due in 5 years without grace at 8% interest payable annually, and pays the banker a bonus of $500 in cash for making the loan; what rate per cent. does he pay?

I. Solution by ROBERT J. ALEY, A. M., Professor of Mathematics in the Indiana University, Bloomington, Indiana.

Simple interest on $5000 for 5 years at 8% is $2000.

If the interest is paid annually in advance, the loss of the use of the money to the borrower is the interest on $400 for (5+4+3+2+1) years which is $480. If the interest is paid annually but not in advance the loss is the interest on $400 for (4+3+2+1) years which is $320. Hence the total interest paid is

$$\$2000+\$500+\$480=\$2980$$

$$\text{or } \$2000+\$500+\$320=\$2820.$$

Interest for 1 year is $\frac{1}{5}$ of $2980=$596, or $\frac{1}{5}$ of $2820=$564.

$$\$596\div\$5000=11.92\%$$

$$\$564\div\$5000=11.28\%.$$

II. Solution by FRANK HORN, Kidder Institute, Kidder, Missouri.

$$\text{II.}\left\{\begin{array}{rl}
1. & 100\%=\$5000=\text{face of note.}\\
2. & 1\%=\$50.\\
3. & 8\%=8\times\$50=\$400=\text{interest for 1 year.}\\
4. & 5\times\$400=\$2000=\text{interest for 5 years.}\\
5. & \$2500=\$2000+\$500=\text{interest paid including the bonus.}\\
6. & \$4500=\$5000-\$500=\text{amount the borrower kept.}\\
7. & \therefore\ \$2500=\text{the interest on }\$4500\text{ for 5 years.}\\
8. & \$500=\tfrac{1}{5}\text{ of }\$2500=\text{the interest for 1 year.}\\
9. & \$4500=100\%\text{ of itself.}\\
10. & \$1=\tfrac{1}{45}\%.\\
11. & \$500=500\times\tfrac{1}{45}\%=11\tfrac{1}{9}\%.
\end{array}\right.$$